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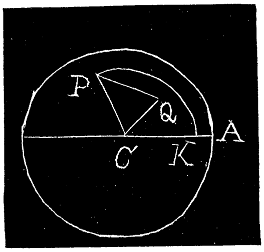
Let $P(x,\ \theta)$ and $Q(y,\ \phi)$ be the random points. Then we have

$$PQ=\sqrt{x^2+y^2-2xy\cos(\theta-\phi)}.$$

Now with C as a center and a radius CP describe the arc PK.

It is evidently necessary to consider only those positions of the two points in which Q is confined to the sector PCK, since under this limitation PQ will take all possible distances.



An element of area of the circle at the point P is $xdxd\theta$, and at the point Q it is $ydyd\phi$.

The limits of y are 0 and x; of x, 0 and r; of ϕ, 0 and θ; and those of θ, 0 and 2π.

Consequently, we have for the required mean distance

$$M=\frac{1}{(\pi r^2)^2}\int_0^{2\pi}\int_0^{\theta}\int_0^{r}\int_0^{x}\sqrt{[x^2+y^2-2xy\cos(\theta-\phi)]}\,xdxd\theta.ydyd\phi$$

$$=\frac{4}{3\pi^2r^4}\int_0^{2\pi}\int_0^{\theta}\int_0^{x}\Big[8\sin^3\tfrac{1}{2}(\theta-\phi)+6\sin^3\tfrac{1}{2}(\theta-\phi)\cos(\theta-\phi)$$

$$-\tfrac{3}{2}\sin^2(\theta-\phi)\cos(\theta-\phi)\log\left(\frac{1+\sin\frac{1}{2}(\theta-\phi)}{\sin\frac{1}{2}(\theta-\phi)}\right)-1+\cos^2(\theta-\phi)\Big]x^4\,d\theta d\phi dx$$

$$=\frac{r}{15\pi^2}\int_0^{2\pi}\int_0^{\theta}\Big[8\sin\tfrac{1}{2}(\theta-\phi)+40\sin\tfrac{1}{2}(\theta-\phi)\cos^2\tfrac{1}{2}(\theta-\phi)$$

$$+6\sin^2(\theta-\phi)\cos(\theta-\phi)\log\left(\frac{1+\sin\frac{1}{2}(\theta-\phi)}{\sin\frac{1}{2}(\theta-\phi)}\right)$$

$$-48\sin\tfrac{1}{2}(\theta-\phi)\cos^4\tfrac{1}{2}(\theta-\phi)+3\cos2(\theta-\phi)-1\Big]d\theta d\phi$$

$$=\frac{2r}{45\pi^2}\int_0^{2\pi}\Big[32-32\cos\tfrac{1}{2}\theta-16\sin^2\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta+24\sin^4\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta+3\sin\theta\cos\theta$$

$$+3\sin^3\theta\log\left(\frac{1+\sin\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}\right)\Big]d\theta=\frac{128r}{45\pi}.$$

Professor Walker also furnished a second very excellent solution.

MISCELLANEOUS.

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91. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

The following sides and area are given for a rational triangle in the table of rational scalene triangles on page 167 of Dr. Halsted's "Metrical Geometry (Boston, 1881), viz.: sides, 21, 61, 65; area, 420. The same sides and area are given in Septimus Tebay's "Mensuration" (London and Cambridge, 1868), in a table on page 113.

The sides of this triangle can not all be correct because they are all *odd*.

Assuming that the *area* given is correct, it is required to determine the error in the sides.

I. Solution by M. A. GRUBER. A. M., War Department, Washington, D. C.

Knowing that every integral rational scalene triangle is the sum or the difference of two integral right triangles having equal altitudes, and knowing that 65 and 61 are hypotenuses of integral right triangles, we find that 65, 63, 16; 65, 60, 25; 65, 56, 33; and 65, 52, 39, are the respective sides of the right triangles having the hypothenuse 65; and that 61, 60, 11, are the sides of the only integral right triangle having the hypothenuse 61.

We observe that the triangles whose respective sides are 65, 60, 25, and 61, 60, 11, have the common leg 60. Taking 60 as the altitude, and finding the *sum* and the *difference* of the two triangles, we obtain two integral rational scalene triangles, sides being 65, 61, 36, and 65, 61, 14. The area of the former is $\sqrt{(81\times16\times20\times45)}=1080$, and of the latter, $\sqrt{(70\times5\times9\times56)}=420$.

$\therefore$ The side of the triangle in question should be 14.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Supposing the sides 61 and 65 to be right, but 21 to be wrong, we may put this side$=2a$, since it must be an even number. Thus, we get $(63+a)(63-a)(a+2)(a-2)=420^2$, whence $a^4-3973a^2=-192276$.

$\therefore\ a=\sqrt{\dfrac{3973\pm3975}{2}}$. Taking the lower sign, we obtain $2a=14$. Thus, the sides are 14, 61, 65. There may possibly be more rational values for the three sides to produce the area 420.

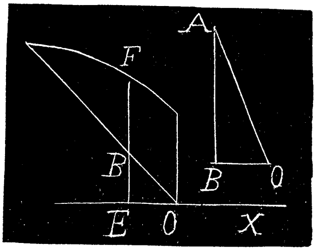
Also solved by *G. B. M. ZERR*.

92. Proposed by J. T. COLE, Columbus, O.

A staff $a=60$ feet high, casting a shadow on a horizantal plane due north $b=20$ feet long, falls due northeast. Find the area covered by the shadow.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let O, B be the same points in both figures. OC the staff after it has fallen to the ground. OA the staff while falling.



Let $\angle AOB=\theta$, altitude of the sun$=\beta$, $OE=x$, $FE=y$. Then $\beta=\tan^{-1}(a/b)=\tan^{-1}(3)$, $AB=a\sin\theta$, $OB=a\cos\theta$, $OE=EB=(a/\sqrt{2})\cos\theta$, $BF=AB\cot\beta$.

$\therefore\ BF=a\sin\theta\cot\beta=b\sin\theta$.

$\therefore\ x=(a/\sqrt{2})\cos\theta$,

$y=x+b\sin\theta=x+(b/a)\sqrt{(a^2-2x^2)}=y'$.

Area passed over by the shadow$=CODFC$.

$$\therefore\ A=\int_0^{a/\sqrt{2}}\int_x^{y'}dxdy=b/a\int_0^{a/\sqrt{2}}\sqrt{(a^2-2x^2)}dx.\quad \therefore\ A=\tfrac{1}{8}\pi ab\sqrt{2}=150\pi\sqrt{2}.$$